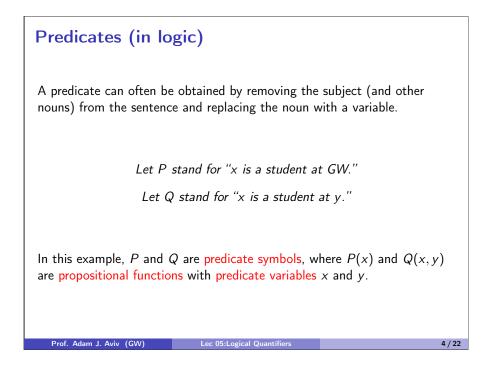


Predicates (grammatically)	
Grammatically, a <i>predicate</i> is the part of the sentence that provides information about the subject (or object) of the sentence.	
John Smith is a student at GW. predicate	
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## Predicates, Domains, and Truth Sets

#### Definition

A predicate is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for those values.

#### Definition

A domain of a predicate variable is the set of all values that may be substituted for a given predicate variable.

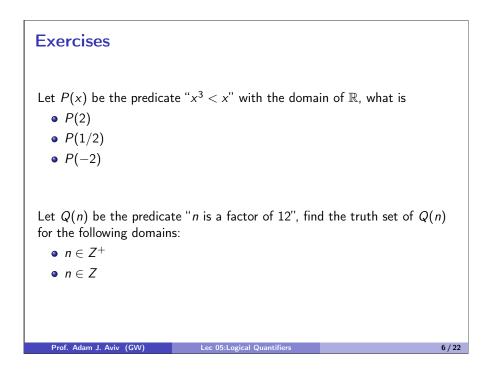
#### Definition

The truth set of predicate P(x) where x has domain D, is the set of all elements in D that generate true statements. Notationally, the truth set of P(X) is

 $\{x \in D \mid P(X)\}$ 

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## Quantifiers

#### Definition

A quantifier is a way to convert a predicate into a statement by declaring how many elements (e.g., some, one, all) may be substituted for the predicate to be true, or put another way, define the size of the truth set for a given domain.

#### Definition

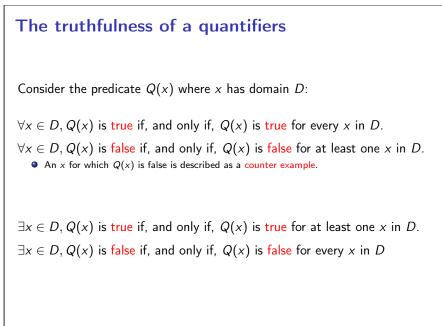
The universal quantifier, denoted with  $\forall$  symbol and read "for all", declares that all elements of a given domain satisfy the predicate.

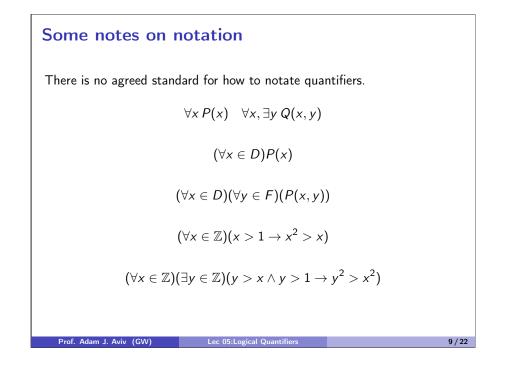
#### Definition

The extensional quantifier, denoted with  $\exists$  symbol and read "exists", declares that there is at least one element of a given domain satisfies the predicate.

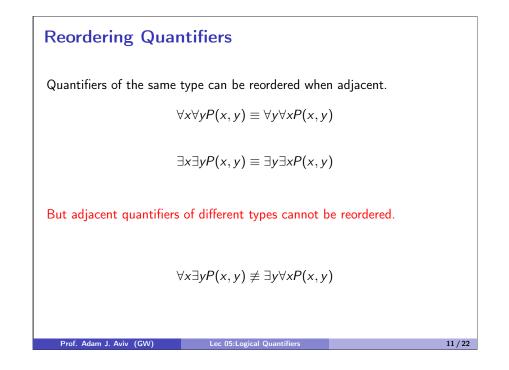
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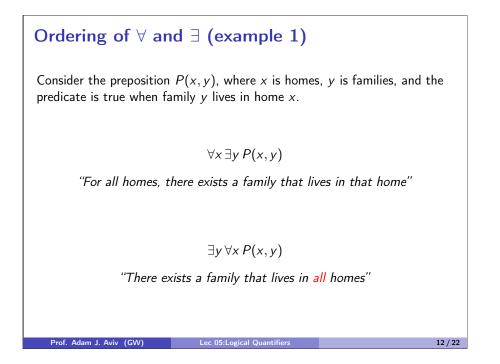
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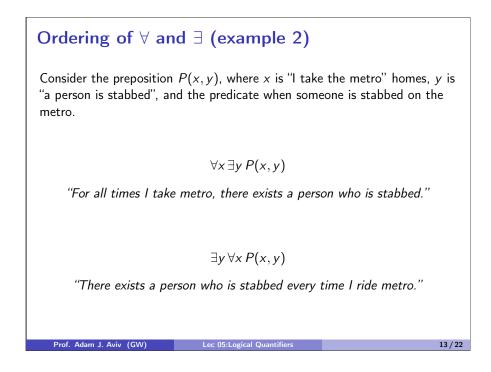




Exercises	
Write out the following statements in plain English, and describe if/why the statements are true or false.	
$(orall x \in \mathbb{Z}^+)(x^2 \geq x)$	
$(\forall w \in \mathbb{R})(w > 2  ightarrow w^2 > 4)$	
$(orall q \in \mathbb{Q})(\exists a, b \in \mathbb{Z})(q = rac{a}{b})$	
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Exercise	
Write the following quantifiers in plain English	
$(orall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x > y)$	
$(\exists y \in \mathbb{Z})(\forall x \in \mathbb{Z})(x > y)$	
$(orall x \in \mathbb{R})(orall y \in \mathbb{R})(\exists a, b \in \mathbb{R})(x = 2/a \wedge y = 2/b)$	
$(\exists y \in \mathbb{Z})(\exists x \in \mathbb{Z})(2x = y)$	
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## **Implicit Quantification**

We can write a statement with an implicit quantification, easily and naturally.

if 
$$x > 2$$
 then  $x^2 > 4$ 

This is equivalent to

$$\forall x \in \mathbb{R}, \text{ if } x > 2 \text{ then } x^2 > 4$$

To indicate an explicit quantification we can use  $\implies$  and  $\iff$  between predicates:

$$x > 2 \implies x^2 > 4$$

The domain of the predicates is assumed to be the same (unless otherwise indicated)

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### Predicative conditional and bi-conditional

#### Definition

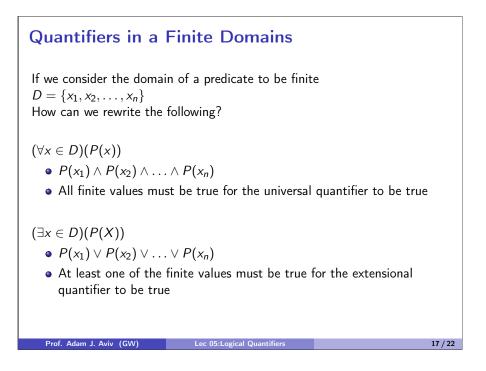
For predicates P(x) and Q(x) with the same domain U, and  $P^t$  and  $Q^t$  are the truth set of P and Q,

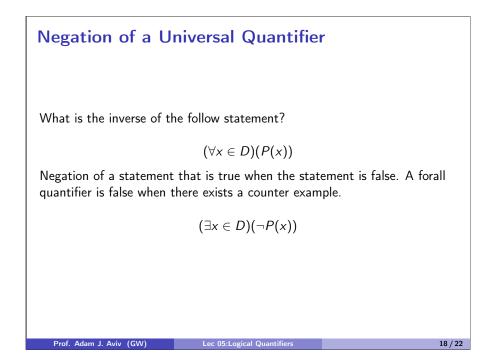
- $P(x) \implies Q(x) \equiv (\forall x \in U)(P(x) \rightarrow Q(x))$ • Every element of P's truth set is in Q's truth set ( $P^t \subseteq Q^t$ )
- $P(x) \iff Q(x) \equiv (\forall x \in U)(P(x) \leftrightarrow Q(x))$ • The truth sets of P and Q are equivalent  $(P^t \subseteq Q^t \land Q^t \subseteq P^t)$

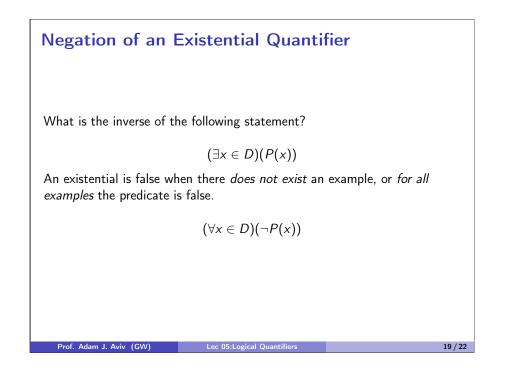
\* We may use and  $\implies$  over  $\rightarrow$  in some cases, despite there not always being an implicit quantification.

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## Negation of Universal Conditional Statement

What is the inverse of the follow statement?

 $(\forall x \in U)(P(x) \rightarrow Q(x))$ 

 $\neg [(\forall x \in U)(P(x) \to Q(x)] \equiv (\exists x \in U)[\neg (P(x) \to Q(x))]$ 

Recall that  $\neg(p 
ightarrow q) \equiv p \land \neg q$ 

$$(\exists x \in U)[\neg (P(x) \rightarrow Q(x))] \equiv (\exists x \in U)(P(x) \land \neg Q(x))$$

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# Connection to DeMorgan's Law

DeMorgan's Law shows us that the inversion of a  $\wedge$  is a  $\lor,$  and the inversion of a  $\lor$  is a  $\wedge$ 

$$eglinetity \neg (p \land q) \equiv \neg p \lor \neg q$$
  
 $eglinetity \neg (p \lor q) \equiv \neg p \land \neg q$ 

Similarly, the inversion of  $\forall$  is an  $\exists$ , and the inversion of a  $\exists$  is a  $\forall$ .

$$\neg(\forall x, P(x)) \equiv \exists x, \neg P(x) \neg(\exists x, P(x)) \equiv \forall x, \neg P(x)$$

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Exercises
Write a logically equivalent statement for the following English sentences
There is at least three distinct integers for which $P(x)$ is true.
There is at most three distinct integers for which $P(x)$ is true.
There is exactly three distinct integers for which the $P(x)$ is true.
There does not exist two distinct integers for which the $P(x)$ is true.
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