

What is a set?

Definition

A set is a well defined collection of objects that are described as members or elements of the set.

Example

The set of the first 5 prime numbers

$$A = \{2, 3, 5, 7, 11\}$$

The numbers 5 is a member of the set A, or $5 \in A$.

Example

The set of odd numbers between 11 and 17, inclusive.

$$B = \{11, 13, 15, 17\}$$

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The number 12 is *not* an element of the Set *B*, or $12 \notin A$.

Set Roster Notation

The set is described by using braces $\{\,\}$ with all elements seperated by commas.

Example

• {red, blue, orange} $\{1\}$ {a, b, d}

You can also specify a set to contain another set using nested braces:

Example

- $\{1, \{2, 3\}\}$
- The set containing 1 and the set {2,3}

We can use ellipses " \dots " (read "as so forth") to indicate sets that continue on infinitely.

Example

- {0,1,2,...} (set of positive integers)
- $\{\ldots, -3, -2, -1\}$ (set of negative integers)

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Set Equality

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Definition

Two sets A and B are equal (written as A = B) when they contain the same elements. (the order of the elements is irrelevant)

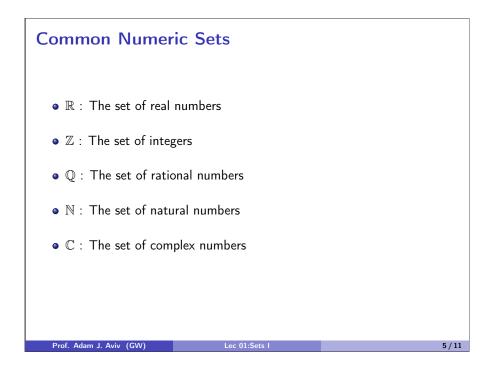
- Is A equal to B?
 - (1) A = {1, 2, 3} B = {1, 2, 3}
 ▶ yes, A = B
- (2) A = {1,2,3} B = {2,1,3}
 ▶ yes, A = B even though elements presented in different orders

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(3) A = {1,2,3} B = {3,3,1,2,2,3,3}
▶ yes, A = B even with multiple repetitions.

(4) A = {1, 2, 3} B = {1, 1, 3} No, A ≠ B because 2 ∈ A but 2 ∉ B

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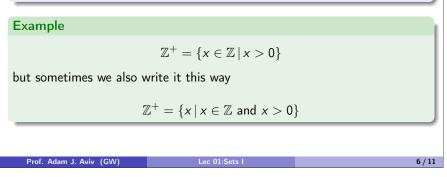
Set Builder Notation

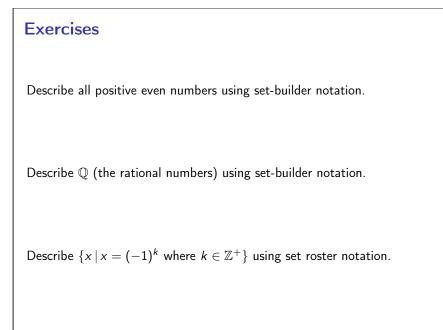
Set Builder Notation

Describe a set where some condition is met.

$$\{x \in S \mid P(x)\}$$

"The set of all elements x in S such that some property/proposition P(x) is true"





Interval Notation

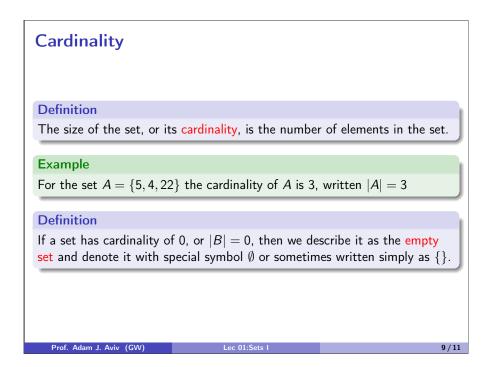
It's common to want to describe an interval within a numeric set, such as $\mathbb Z$ or $\mathbb R.$ This is easily done using the following notation:

$$\{x \in \mathbb{R} \mid a < x < b\}$$

In plain language, this is the set of real numbers between a and b.

However, this is cumbersome, so we have the following shorthand.

$$\begin{array}{ll} (a,b) = \{x \in \mathbb{R} \mid a < x < b\} & [a,b] = \{x \in \mathbb{R} \mid a \le x \le b\} \\ (a,b] = \{x \in \mathbb{R} \mid a < x \le b\} & [a,b) = \{x \in \mathbb{R} \mid a \le x < b\} \\ (a,\infty) = \{x \in \mathbb{R} \mid x > a\} & [a,\infty) = \{x \in \mathbb{R} \mid x \ge a\} \\ (-\infty,a) = \{x \in \mathbb{R} \mid x < a\} & (-\infty,a] = \{x \in \mathbb{R} \mid x \le a\} \end{array}$$





Can you construct an argument to show that $\{\}=\emptyset$ based on the equality rule from before?

Recall that "two sets A and B are equal when they contain the same elements"



What is $|\{\{\}, 3, \emptyset, \{1, 2, 3\}\}|$?

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If two sets A and B have the same cardinality, |A| = |B|, is it the case that A = B?

If two sets A and B are equal, A = B, is it the case that they have the same cardinality, |A| = |B|?

